[This question paper contains 4 printed pages.]

Your Roll No. 2022

Sr. No. of Question Paper : 2792 **A**

Unique Paper Code : 62354443

Name of the Paper : Analysis (LOCF)

Name of the Course : **B.A. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.

1. (a) Find the supremum and infimum of the following sets, if they exist

(i) $E=\left\{1+\frac{(-1)^n}{n} : n \in N\right\}$

(ii) $F=\left\{2, \frac{3}{2}, \frac{4}{3}, \ldots, \frac{n+1}{n}, \ldots\right\}$

P.T.O.

(b) State Sequential criterion of continuity.

Define $f: \mathbb{R} \to \mathbb{R}$ by $f(x) = \begin{cases} 1, & when\ x\ is\ rational \\ 0, & when\ x\ is\ irrational \end{cases}$.

Show that f is discontinuous on $\mathbb{R}$.

(c) Give an example of a non-empty bounded subset S of R whose supremum and infimum both belong to R ~ S.

(d) Test for convergence the series

$$\sum_{n=1}^{\infty} \left(\sqrt{n^4+1} - \sqrt{n^4-1}\right)$$

2. (a) Show that $f(x) = \begin{cases} \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

is discontinuous at $x = 0$

(b) State Archimedean Property of real numbers. Use it to prove that if t > 0, there exists $n_t \in N$ such that $0 < 1/n_t < t$.

(c) Show that the function $f(x) = x^2$ is uniformly continuous on]–2, 2[.

(d) Prove that if

$$a_n = \frac{1}{n}\{(n+1)(n+2).....(n+n)\}^{1/n}$$

then $\langle a_n \rangle$ converges to $\frac{4}{e}$.

3. (a) Prove that every Cauchy sequence is bounded but converse need not be true.

(b) Prove that the series

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots\ldots \text{ converges.}$$

(c) Show that the sequence $\langle s_n \rangle$ where $S_n=\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{n!}$ is convergent.

(d) Show that $\lim\limits_{n\to\infty}\frac{1+3+5+\ldots\ldots+(2n-1)}{n^2}=1$.

4 (a) Show that the series $1+r+r^2+r^3+\ldots.(r>0)$ converges if $r<1$ and divergence if $r\geq 1$.

(b) Show that the sequence $\langle a_n \rangle$ where $a_n=1+\frac{1}{3}+\frac{1}{3^2}+\ldots+\frac{1}{3^{n-1}}$ converges. Find $\lim\limits_{n\to\infty} a_n$?

(c) Test for convergence the series

$$\frac{x}{2\sqrt{3}}+\frac{x^2}{3\sqrt{4}}+\frac{x^3}{4\sqrt{5}}+\ldots\ldots(x>0)$$

(d) Show that the series $\sum\limits_{n=1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ is convergent.

5 (a) Define Alternating series of real numbers. Test for the convergence and absolute convergence of the series.

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots.$$

(b) Prove that every continuous function is integrable.

P.T.O.

(c) Define a conditionally convergent series. Test for the convergence and absolute convergence of the series.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n+2}{2^n+5}.$$

(d) Test for convergence the series

$$\sum_{n=1}^{\infty} \frac{1.3.5.....(2n-1)}{2.4.6.....2n} \cdot \frac{1}{n}$$

6 (a) Define Riemann integrability of a bounded function f on a bounded closed interval $[a, b]$. Show that the function f defined on $[a, b]$ as

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

is not Riemann integrable.

(b) Test for convergence the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cos n\alpha}{\sqrt{n^3}}, \alpha \text{ being real.}$$

(c) Integrate the function $f(x) = x[x]$ on $[0, 4]$, where $[x]$ denotes the greatest integer not greater than x.

(d) Show that the sequence $\langle a_n \rangle$ defined as $a_n = 1 + \frac{1}{6} + \frac{1}{11} + ... + \frac{1}{5n-4}$ is not a Cauchy sequence.

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